MATHEMATICAL MODEL OF A UNIFORMLY FLUIDIZED BED — INTRODUCTION

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Basical relations among geometrical characteristics of studied arrangements of particles in the bed are expressed by use of the theoretical model published earlier. One of these until now unknown arrangement corresponds to a uniformly fluidized bed at small values of Ar number. It is demonstrated that porosity of a uniformly fluidized bed at incipient fluidization can supply under certain conditions, important information on geometrical structure of the bed.

The uniformly fluidized bed has, according to the theoretical model[1], for any porosity, the particles arranged in one of the structure type A_1, A_2, A_x or B_1, B_2, B_3. Structures of these types represent a possible arrangement of particles in the bed through which the fluid flows at the equilibrium of forces acting on each particle. Structures of the type A_1, A_2, A_x have particles and boundary flow patterns in main planes situated

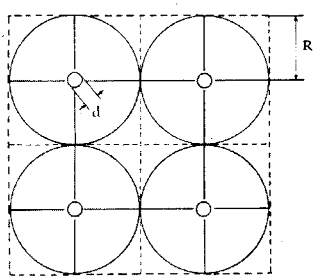

FIG. 1

Surroundings of Particle Boundaries and Boundary Flow Patterns with Particles Situated in Edges of Squares where the Boundary Flow Pattern Is a Square

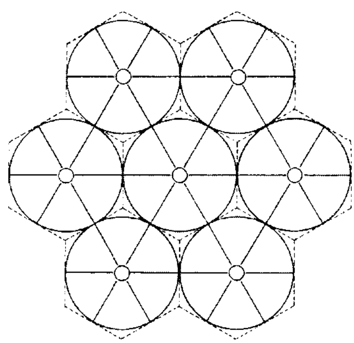

FIG. 2

Surroundings of Particles and Distribution of the Main Plane to Sub-Spaces of the Flow with Boundary Circles when the Boundary Flow Patterns Are Uniform Hexagons and Particles are Situated in Edges of Equilateral Triangles according to Figs 1–3. Into the dashed boundary flow patterns are inscribed the boundary circles with spherical particles in their centres. Elementary particle layers in the main planes are situated one after another along the height of the bed so, that the position of particles at a distance h (distance of the main planes) exactly repeats. All the layers determined by the main planes can become identical only by their vertical displacement. Structures of the type B have the particles arranged in the main planes according to Fig. 1 or 2. Elementary particle layers in the main planes are situated one after another in the vertical direction so that particle centres in the next main plane are exactly above the centres of symmetry of a constant velocity regions in the preceding main plane. The structure B_1 then corresponds to Fig. 1; boundary circles in two consecutive main planes of B_1-type structure are drawn in Fig. 4. Position of boundary circles in the third principal plane is obtained by a vertical shift of the first plane and the situation repeats. Centres of particles which are not indicated are identical with the centres of boundary circles. Boundary flow patterns are plotted only in the first main plane. Structures of the type B_2 or B_3 correspond to Fig. 2. In Fig. 5 are plotted boundary circles in two consecutive main planes which are characteristic for the structure of type B_2. Position of boundary circles

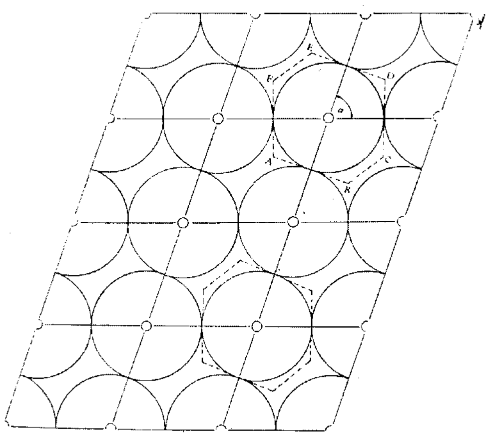

FIG. 3

Surroundings of Particles and Distribution of the Main Plane to Sub-Spaces of the Flow with Boundary Circles when the Flow Patterns are Hexagons ABCDEF and Particles are Situated in Edges of Rhombs with a Sharp Angle $\alpha \in (\pi/3, \pi/2)$

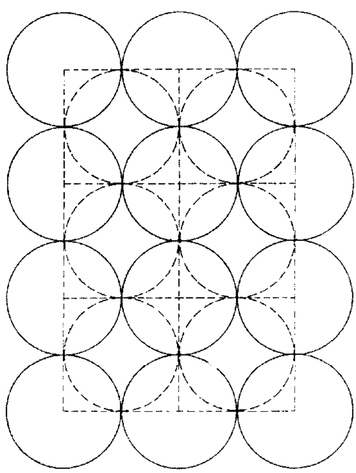

FIG. 4

Mutual Situation of Main Planes for Structures of Type B_1

in the third main plane is obtained by vertical shift of the first plane and the situation repeats. Boundary flow patterns are plotted only in the first main plane. Fig. 6 is characteristic for the structure of type B_3 where the boundary flow patterns are plotted only in one plane. Position of boundary circles in the fourth main plane is obtained by vertical shift of the first plane and the situation repeats.

ELEMENTARY CELLS OF VARIOUS STRUCTURES

For each of the aforementioned type structures, the elementary cell can be defined as a spacial formation having the following characteristics:

a) by arrangement of elementary cells can be formed a whole unlimited bed with arbitrary porosity and without overlaps of voids.

b) the elementary cell has as a base in the main plane an area limited by the boundary flow pattern.

c) each elementary cell has the same particle mass and the porosity of the elementary cell equals to the bed porosity.

For the mathematical model, following data on elementary cell are of importance:

a) The following quantities expressed by use of the radius R – of the boundary circle: The lengths $a_1, a_2, \ldots$ denoting sides of the boundary flow pattern, area S_1

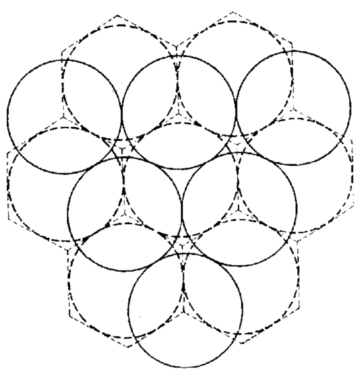

FIG. 5

Mutual Situation of Main Planes for Structures of Type B_2

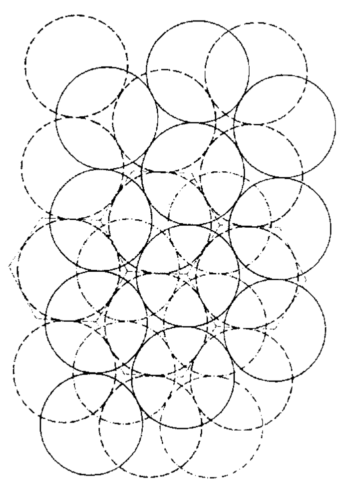

FIG. 6

Mutual Situation of Main Planes for Structures of Type B_3

limited by the boundary flow pattern, area A in the boundary flow pattern on the external side of the boundary circle, area A^* in the boundary flow pattern across which the fluid flows around the particle.

b) The cell volume V_B given by use of the radius R of the boundary circle and by use of the distance h of the main planes.

c) In a uniformly fluidized bed, to one particle corresponds the bed volume V_0 given by relation

$$V_0 = \pi d^3/6(1 - \varepsilon)\,. \qquad (1)$$

When $V_B = F(R, h)$ and the cell contains a mass corresponding to N_0 particles, we can write

$$F(R, h) = N_0 \pi d^3/6(1 - \varepsilon)\,. \qquad (2)$$

In our preceding study[1] we have introduced an unknown function $\varphi(\varepsilon)$, which is defined by the relation

$$h/R = \varphi(\varepsilon)\,. \qquad (3)$$

Function F in relation (2) is of such character that from Eqs (2) and (3) can be expressed $R = dF_1[\varepsilon, \varphi(\varepsilon)]$; $h = dF_2[\varepsilon, \varphi(\varepsilon)]$.

Definition of elementary cells and their characteristics. Structures of type A: An elementary cell can have as its base a boundary flow pattern which is either a square (structures A_1, $a = 2R$) as in Fig. 1, or a regular hexagon (structure A_2, $a = 2R(3)^{1/2}/3$), as in Fig. 2 or a hexagon (structure A_x, $a_1 = a_3 = a_5 = 2R(\cos\alpha)/[\cos(\alpha/2)]$, $a_2 = a_4 = a_6 = 2R\,\mathrm{tg}(\alpha/2)$) as in Fig. 3. For the height of elementary cells can be taken in all cases the distance of the main planes h. Elementary cell will contain one half from each of two spheres, *i.e.* the mass of one sphere and then it holds $V_B = V_0 = S_1 h$; $N_0 = 1$.

Structures of type B: An elementary cell can also have as its base the boundary flow pattern which is either a square (structures B_1, $a = 2R$) as in Fig. 1 or a regular hexagon (structures B_2, B_3, $a = 2R(3)^{1/2}/3$) as in Fig. 2. It is advantageous for the square base to choose the height of the cell equal to the distance of the main planes h. Then, according to Fig. 4, the cell can have on its base one half of one sphere and under the wall opposite to the base one eighth of four spheres each or *vice versa*. The diagram of particle distribution repeats in the vertical direction in each third cell. Each cell has the particle mass equal to one sphere. It holds $V_0 = V_B = S_1 h$; $N_0 = 1$.

If the particles are arranged in the main plane according to Fig. 2, the elementary layers in the main planes can be set vertically as shown in Fig. 5 (structures B_2) or Fig. 6 (structures B_3). In case of the type B_2 let us choose for the cell height (whose base is a regular hexagon of the boundary flow pattern) the distance of the main planes h. On the base in the elementary cell is one half of one sphere and under the wall opposite to the base is one sixth of three spheres each or *vice versa*. The pattern of particle distribution repeats in the vertical direction after two cells. The mass of each cell equals to one sphere, *i.e.* $V_0 = V_B = S_1 h$; $N_0 = 1$.

The base of a cell for structures B_3 can also be the regular hexagon of the boundary flow pattern. Let us choose for better illustration the height of the cell as $2h$. One main plane is denoted as the zero plane and on the base of the elementary cell in this main plane is situated one half of one sphere. In the first adjoining (above located) main plane is one third of three spheres each and under the second main

TABLE I

Basical Data on Elementary Cells for Various Types of Structures

Quantity	A_x	A_1, B_1
a	$a_1 = a_3 = a_5 = 2R (\cos \alpha)/[\cos (\alpha/2)]$ $a_2 = a_4 = a_6 = 2R \operatorname{tg} (\alpha/2)$	$2R$
A	$R^2\{4 \operatorname{tg} (\alpha/2) + (\sin 2\alpha)/[\cos^2 (\alpha/2)] - \pi\}$	$R^2(4 - \pi)$
A^*	$R^2\{4 \operatorname{tg} (\alpha/2) + (\sin 2\alpha)/[\cos^2 (\alpha/2)]\} - \pi d^2/4$	$4R^2 - \pi d^2/4$
h	$d\left[\dfrac{\pi}{6\left(4 \operatorname{tg} (\alpha/2) + \dfrac{\sin 2\alpha}{\cos^2 (\alpha/2)}\right)(1 - \varepsilon)\,\varphi(\varepsilon)}\right]^{1/3} \varphi(\varepsilon)$	$d\left[\dfrac{\pi}{24(1 - \varepsilon)\,\varphi(\varepsilon)}\right]^{1/3} \varphi(\varepsilon)$
N_0	1	1
R	$d\left[\dfrac{\pi}{6\left(4 \operatorname{tg} (\alpha/2) + \dfrac{\sin 2\alpha}{\cos^2 (\alpha/2)}\right)(1 - \varepsilon)\,\varphi(\varepsilon)}\right]^{1/3}$	$d\left[\dfrac{\pi}{24(1 - \varepsilon)\,\varphi(\varepsilon)}\right]^{1/3}$
S_1	$R^2\{4 \operatorname{tg} (\alpha/2) + (\sin 2\alpha)/[\cos^2 (\alpha/2)]\}$	$4R^2$
V_B	$R^2\{4 \operatorname{tg} (\alpha/2) + (\sin 2\alpha)/[\cos^2 (\alpha/2)]\}\, h$	$4R^2 h$
hR^2	$\dfrac{\pi d^3}{6\{4 \operatorname{tg} \alpha/2 + (\sin 2\alpha)/[\cos^2 (\alpha/2]\}(1 - \varepsilon)}$	$\dfrac{\pi d^3}{24(1 - \varepsilon)}$

plane one sixth of three spheres each. The cell above this main plane extends from the second to the fourth main plane. Above the second main plane (the base) is one sixth of three spheres each, on the third is one whole sphere and under the fourth main plane is one sixth of three spheres each. The third cell extends from the fourth to the sixth main plane. Above the fourth main plane is one sixth of three spheres each, on the fifth one third of three spheres each and under the sixth main plane is one half of one sphere. The fourth cell in the vertical direction has already the same distribution of particles as the first one. The mass of each cell equals to two spheres, *i.e.* $2V_0 = V_B = 2S_1h$, $N_0 = 2$, but the height of the cell is $2h$.

Basical data on elementary cells are given in Table I. For structures A_x, α represents the angle as in Fig. 3, where $\alpha \in (\pi/3, \pi/2)$. The analysis shows that function $y = 4 \operatorname{tg}(\alpha/2) + (\sin 2\alpha)/[\cos^2(\alpha/2)]$ is in the interval $\alpha \in < \pi/3, \pi/2 >$ increasing and reaches the value $y \in \langle 2(3)^{1/2}, 4 \rangle$, *i.e.* the data given in Table I for structures A_1, A_2, B_1, B_2, B_3 represent the limiting values for structures A_x.

TABLE I
(*Continued*)

A_2, B_2	B_3
$2R(3)^{1/2}/3$	$2R(3)^{1/2}/3$
$R^2[2(3)^{1/2} - \pi]$	$R^2[2(3)^{1/2} - \pi]$
$2R^2(3)^{1/2} - \pi d^2/4$	$2R^2(3)^{1/2} - \pi d^2/4$
$d\left[\frac{\pi}{12(3)^{1/2}(1-\varepsilon)\varphi(\varepsilon)}\right]^{1/3}\varphi(\varepsilon)$	$d\left[\frac{\pi}{12(3)^{1/2}(1-\varepsilon)\varphi(\varepsilon)}\right]^{1/3}\varphi(\varepsilon)$
1	2
$d\left[\frac{\pi}{12(3)^{1/2}(1-\varepsilon)\varphi(\varepsilon)}\right]^{1/3}$	$d\left[\frac{\pi}{12(3)^{1/2}(1-\varepsilon)\varphi(\varepsilon)}\right]^{1/3}$
$2R^2(3)^{1/2}$	$2R^2(3)^{1/2}$
$2R^2h(3)^{1/2}$	$4R^2h(3)^{1/2}$
$\frac{\pi d^3}{12(3)^{1/2}(1-\varepsilon)}$	$\frac{\pi d^3}{12(3)^{1/2}(1-\varepsilon)}$

Porosity at Incipient Fluidization and Structure of the Uniformly Fluidized Bed

Important information on the geometrical structure of the uniformly fluidized bed and of flow properties in such a bed can be obtained by use of a suitable mathematical model based on same experimental results concerning a uniformly fluidized bed in the region at incipient fluidization.

It has been found experimentally[2] that under a sufficiently slow continuous decrease of the superficial velocity of the fluid in uniformly fluidized bed of spherical particles, the bed height continuously decreases until the state of a fixed bed is reached. Porosity of this fixed bed is $\varepsilon_0 = 0{\cdot}412 \pm 0{\cdot}004$ and at further decrease of the velocity it does not vary. At porosity $\varepsilon_p = 0{\cdot}420 \pm 0{\cdot}004$ the visible vibration of particles in the bed stops. Values ε_0 and ε_p are independent of Ar number and the smallest value of Ar number in the experimental serie was Ar = 17. Corresponding pairs w, ε at Ar = constant as determined experimentally at increasing and decreasing velocity w in the region at incipient fluidization were practically consistent. The same experiments were repeated by Neužil and Hrdina[3]. These authors have determined that ε_0 depends only on the ratio d/D and have presented the relation

$$\varepsilon_0 = 0{\cdot}404 \pm 0{\cdot}429 d/D \,. \qquad (4)$$

Continuous change of the bed porosity up to the value ε_0 and the independence of corresponding pairs w, ε at Ar = const. with the decreasing or increasing superficial velocity are of a special importance. They proved that when at $\varepsilon > \varepsilon_0$ there act the friction forces among the particle surfaces, their effect continuously increases with the decreasing porosity or decreases continuously with the increasing porosity and that for certain particles they depend only on the bed porosity $\varepsilon > \varepsilon_0$. The bed expansion is characterized by the expansion equation Re = f(Ar, ε), which at small Ar values has such a form that its log Re ~ log ε plot at Ar = const. is a straight

Table II

Minimum Porosity ε_{min} for Various Structures when $2R = d$ and $h/R = \vartheta$

Quantity	A_1	A_2	A_x
R	$d/2$	$d/2$	$d/2$
ϑ	2	2	2
ε_{min}	0·4764	0·3954	$1 - \dfrac{2\pi}{3\{4\,\mathrm{tg}\,(\alpha/2) + (\sin 2\alpha)/[\cos^2(\alpha/2)]\}}$

line. As soon as the particle friction begins to affect significantly the equilibrium of forces with decreasing porosity, then the deviation of experimentally determined ε from those corresponding to this straight line should simultaneously appear. This deviation should be directed to the region of greater ε and should increase with the decreasing particle diameter. The experiments performed by the author eliminates deviations of this kind. From this results that at porosities $\varepsilon > \varepsilon_0$, only gravity, drag and buoyancy forces affect the equilibrium of forces acting on the particle in a uniformly fluidized bed. If we take into consideration eventual systematic errors in the measurement, then in agreement with the aforesaid the porosity at incipient fluidization has a value ε_0 from the interval

$$\varepsilon_0 \in \langle 0{\cdot}404;\ 0{\cdot}416 \rangle\ . \qquad (5)$$

At this value ε_0 the fixed bed obtained by a sufficiently slow decrease of the superficial liquid velocity has the same particle arrangement as the uniformly fluidized bed closely before the fluidized state ceases. The fluidized state ceases to exist (*i.e.* the drag force becomes smaller than the resulting force of gravity and buoyancy) when particles are in mutual contact. Let us imagine that we slowly, continuously decrease the superficial fluid velocity in a uniformly fluidized bed and at a certain value w_{min} (to which corresponds the porosity ε_{min}) the drag force on the particle beginsfto be smaller than the force resulting from buoyancy and gravity. From the first condition of stability of beds with structures A or B implies that at $w \leqq w_{min}$ there does not exist any arrangement of particles in which the equilibrium of force could be renewed. At $w \leqq w_{min}$ the equilibrium of forces can persist only as a consequence of interactions among particles across their contacting surfaces. Then, if the bed porosity at $w < w_{min}$ decreases continuously with the decreasing velocity w, the friction forces must act simultaneously. Geometrical arrangement of particles in the bed at $\varepsilon < \varepsilon_{min}$ can differ in such cases from structures A and B

TABLE II
(*Continued*)

B_1	B_2	B_3
$d/2$	$d/2$	$d/2$
$2^{1/2}$	$2(2/3)^{1/2}$	$2(2/3)^{1/2}$
0·2595	0·2595	0·2595

unlike of states at $\varepsilon > \varepsilon_{min}$. From properties of the expansion equation results that in a uniformly expanded bed at $\varepsilon > \varepsilon_0$ the friction of particles does not appear, so the fixed bed obtained by a slow continuous decrease of the fluid velocity at ε_0 has some of the structures A or B. But we consider, also the cases when this conclusion is not valid.

Uniformly fluidized bed can change into the fixed bed with porosity ε_0 in different ways:

a) Table II gives minimum porosities ε_{min} which would correspond at the particle arrangement according to individual structures A or B to the state of the bed when the particles in the main plane are in contact *i.e.* where $2R = d$ and at the same time the ratio h/R has such value ϑ that also the elementary particle layers in main planes are in contact. Values ε_{min} for the structures A_x are in this case between the values ε_{min} for structures A_2 and A_1. As can be seen, value $\varepsilon_{min} = 0{\cdot}2595$ for structures B_1, B_2, B_3 is in these cases in an obvious discrepance with the experimentally found condition (5), *i.e.* $\varepsilon_{min} \neq \varepsilon_0$. Fixed beds in structures A_1, A_2, A_x are unstable arrangements where each sphere is supported from the bottom only at one point. Martin and coworkers[4] have found experimentally that the fixed bed for structure A_1, where $\varepsilon_{min} = 0{\cdot}4764$ is very unstable and can exist only in a vessel with square cross-section and precisely kept dimensions, and if the spheres are practically ideal and very accurately equal in size.

If the assumed contact of particles takes place for the structure A_1 in a cylindrical vessel simultaneously with the cease of the fluidized state, then with further continuous decrease of velocity a fixed bed would form with the porosity $\varepsilon_{min} < 0{\cdot}4764$ which could satisfy the condition (5). However, this to be possible, for the structure A_1, the condition must be met

$$h/R = \vartheta = 2\,, \quad \text{if} \quad \varepsilon = 0{\cdot}4764\,. \tag{6}$$

b) When the fluidized state ceases the particles come into contact only in main planes but elementary particle layers in main planes are not in contact (*i.e.* in the vertical direction). Then it simultaneously holds

$$2R = d\,, \quad h/R > \vartheta\,. \tag{7}$$

Such beds are very unstable and can exist only when they are suitably surrounded by a solid wall. Stability of the fixed bed which forms at a slow, continuous decrease of the liquid velocity is good. Thus it can be stated, that the particle arrangement in a uniformly fluidized bed at $\varepsilon \to \varepsilon_0$ differs under condition (7) from the particle arrangement for structures A, B if $\varepsilon_{min} = \varepsilon_0$ holds simultaneously. Nevertheles in Fig. 7 is plotted the respective dependence $\varepsilon_{min} = F_0(h/R)$ for all types of structures

A, B in order to enable its exact smoothening. Structures of the type A_x have values ε_{min} between of those for structures A_1, B_1 and A_2, B_2, B_3). Necessary relations $\varepsilon_{min} = F_0(h/R)$ can be obtained by writting the relation for hR^2 according to Table I and from it there is explicitly expressed h/R .Into the right hand side is substituted for $R = d/2$ and for the value ε the respective value ε_{min}. It obviously holds $\varepsilon_{min} > \varepsilon_0$.

c) In structures A the fluidized state can cease at the formation of the fixed bed when it simultaneously holds

$$h = d\,, \quad 2R = a_0 d\,, \tag{8}$$

where the coefficient $a_0 > 1$. In such case the particles are not in contact in the main plane. Fixed beds of this type are very unstable. If the equilibrium of forces on the particle is upset at porosity ε_{min}, the bed would not become fixed at this value, but would steady at same smaller porosity and at other structure than are those A or B. If this case would materialize, then between h/R and ε_{min} the following relations should hold:

For structures A_1, $\varepsilon_{min} = 1 - \pi/6a_0^2$, and $h/R = 2/a_0$.
Thus

$$h/R = [24(1 - \varepsilon_{min})/\pi]^{1/2}\,. \tag{9}$$

Similarly, for structures A_2 holds

$$h/R = [12(3)^{1/2}(1 - \varepsilon_{min})/\pi]^{1/2}\,. \tag{10}$$

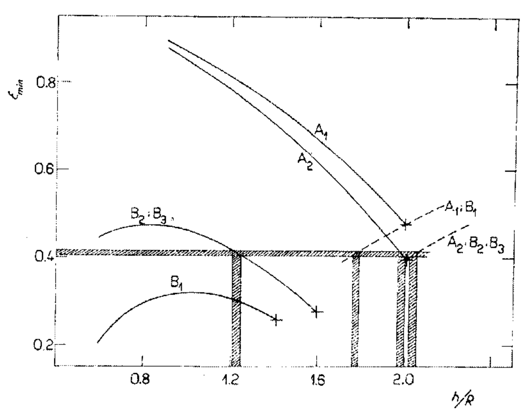

FIG. 7

Porosity ε_{min} (at which the fluidization could cease to exist at the simultaneous contact of particles) in Dependence on Ratio $h/R = \varphi(\varepsilon)$

If for the given ε_{min} according to Eq. (9) $h/R = [h/R]_1$ and according to Eq. (10) $h/R = [h/R]_2$, then to structures A_x correspond the values $h/R \in ([h/R]_2; [h/R]_1)$. It is obvious that $\varepsilon_{min} > \varepsilon_0$.

d) Structures of the type B can change the fluidized state into the relatively stable fixed bed when the relations hold simultaneously

$$h/R \in (0, \vartheta),\ 2R = a_0 d\,, \tag{11}$$

where $a_0 > 1$. Elementary particle layers from adjoining main planes then come into contact but particles in the main plane are not in contact. Each particle is supported from the bottom by four (B_1) or by three particles (B_2, B_3). The distance of main planes h, particle diameter d and the factor a_0 are related by a unique relation.

For structure B_1 holds: $h = d(1 - a_0^2/2)^{1/2}$; $a_0^2 = 4/[(h/R)^2 + 2]$, or

$$\varepsilon_{min} = 1 - \frac{\pi[(h/R)^2 + 2]^{3/2}}{24(h/R)}\,. \tag{12}$$

Similarly for structures B_2 and B_3 the relations hold:
$h = d(1 - a_0^2/3)^{1/2}$; $a_0^2 = 4/[(h/R)^2 + 4/3]$, or

$$\varepsilon_{min} = 1 - \frac{\pi[(h/R)^2 + 4/3]^{3/2}}{12\sqrt{3}\,(h/R)}\,. \tag{13}$$

It is obvious that the relation $\varepsilon_{min} = \varepsilon_0$ can hold.

We have considered all possibilities at which the fluidized state ceases with the formation of the fixed bed under mutual contact of particles in structures A and B. Results of this analysis are plotted in Fig. 7 with types of structures denoted. Crosses represent fixed beds determined by condition *a*). Dependence $\varepsilon_{min} = F_0(h/R)$ for beds sub condition *b*) is plotted in dashed-lines. Full-lines denoted A_1, A_2, or B_1, B_2 and B_3 correspond with condition *c*), or *d*). Horizontal cross-hatched zone represents the interval by condition (5), *i.e.* $\varepsilon_{min} = \varepsilon_0 \in \langle 0{\cdot}404; 0{\cdot}416 \rangle$. If it intersects the curve $\varepsilon_{min} = F_0(h/R)$ for the structure of any type, then the bed in the region at incipient fluidization can have the respective structure, if simultaneously the ratio h/R has the required value. Thus into account can be taken structures B_2, B_3 when there originates the stable fixed bed as sub condition *d*); structures A_2 and partially also A_x when primarily originate very unstable fixed beds as sub *c*); structures A_1, B_1 or A_2, B_2, B_3 and therefore also structures A_x when primarily originate very unstable fixed beds as sub *b*). In agreement with what has been said the cease of fluidization in conditions sub *b*) and *c*) there is a great probability that a uniformly fluidized bed has in the region at incipient fluidization the structure B_2 or B_3. Required intervals of values h/R which correspond according to Fig. 7 to condition (*5*), are here limited by vertical cross-hatched zones. More accurate informations on these values h/R can be obtained from Table III where is also smoothened a wider range of values ε_{min} than corresponds to condition (*5*).

When considering Fig. 7 and Table III it is necessary to realize that if the equilibrium of forces is upset at incipient fluidization according to conditions *b*) or *c*), it takes place at a greater porosity than given by condition (*5*). Unstability of such unbalanced bed leads to its collapse accompanied by decreased porosity down to the measured value from the interval ε_0 according to (*5*). These cases have been, however, eliminated by considerations including the bed characteristics at incipient fluidization. If the mentioned characteristics are not considered sufficiently convincing, then it can be assumed that fluidization ceases under described conditions sub *b*) and sub *c*). On basis of an accurate observation of the uniformly fluidized bed at $\varepsilon = 0{\cdot}55$, it can be stated that particles are no more in contact. That means that if the fluidization ceases under condition sub *c*), or sub *b*), then in the interval $\varepsilon \in \langle 0{\cdot}404; 0{\cdot}55)$ the porosities must correspond to values $h/R > 1{\cdot}73$. If the uniformly fluidized bed at incipient fluidization has the structure B_2 or B_3, then in the interval, according to condition (*5*), must exist porosities with values h/R according to Eq. (*13*) that are from the interval $h/R \in \langle 1{\cdot}2083; 1{\cdot}2504 \rangle$.

TABLE III

Ratios h/R which Would Correspond to Chosen ε_{min} when the Hydrodynamic Force on the Particle has Just Decreased Below the Sum of Gravity and Buoyancy Forces According to Conditions Given in This Paper sub *b*), *c*) and *d*)

	h/R for types of structure			
ε_{min}	B_2, B_3	A_2	A_1, B_1	A_2, B_2, B_3
	d	*c*	*b*	*b*
0·400	1·2638	1·9924	1·7453	2·0153
0·404	1·2504	1·9857	1·7570	2·0289
0·408	1·2367	1·97905	1·7689	2·0426
0·412	1·2227	1·97235	1·7809	2·0565
0·416	1·2083	1·9656	1·7931	2·0705
0·420	1·1935	1·9589	1·8055	2·0848
0·424	1·1783	1·9521	1·8181	2·0993
0·430	1·1546	1·9419	1·8372	2·1214
0·450	1·0644	1·9076	1·9040	2·1593
0·500	—	1·8188	2·0944	2·4184

From the made considerations results that if we determine for the uniformly fluidized bed by some independent method the values h/R at porosities close to the incipient fluidization, we can on the basis of Fig. 7 significantly limit the number of structures which – according to the theoretical model[1] – come into account for the uniformly fluidized bed in the region at incipient fluidization. From this the structure of the bed can be deduced in the whole range of porosities. In our next study we present a simple mathematical model for calculation of values h/R from experimental data which can be easily obtained on corresponding pairs of superficial velocities and porosities in the uniformly fluidized bed.

LIST OF SYMBOLS

a	side of a uniform n-angle as a boundary flow pattern
a_0	factor from relation $2R = a_0 d$, where $a_0 > 1$
$a_1, \ldots, a_6$	sides of the hexagon in Fig. 3
A_1, A_2, A_x	type of structure
A	area in the boundary flow pattern on the external side of the boundary circle, see Table I
A^*	area in the boundary flow pattern through which the fluid flows around the particle
$\text{Ar} = gd^3(\varrho_s - \varrho_f)\,\varrho_f/\mu^2$	Archimedes number
B, B_1, B_2, B_3	type of structure
d	diameter of spherical particle
D	diameter of cylindrical column
h	distance of main planes
N_0	number of particles to which is equivalent the mass of particles in the elementary cell
R	radius of the boundary circle (inscribed into the boundary flow pattern)
$\text{Re} = wd\varrho_f/\mu$	Reynolds number
S_1	area limited by the boundary flow pattern
V_0	volume of bed corresponding to one particle
V_B	volume of an elementary cell
w	superficial fluid velocity, *i.e.* the volumetric flow rate divided by the area of horizontal cross section through the bed
ε	porosity of a uniformly fluidized bed or of a bed with some other structure of type A or B
ε_0	bed porosity at incipient fluidization of a uniformly fluidized bed, *i.e.* of a fixed bed which forms from the uniformly fluidized bed at slow decrease of superficial fluid velocity
ε_{min}	porosity of the bed with a structure of an arbitrary type A or B which corresponds to the state when the particle just come into contact at least in the horizontal or vertical direction and at a continuing decrease of the fluid velocity at the structure of the same type the equilibrium of forces on the particle would be upset (of gravity, buoyancy and drag)
ε_p	porosity of the uniformly fluidized bed at which stops the visually observable vibration of particles when the fluid velocity is continuously decreased

ϑ	value h/R for individual structures when the particles come so close to each other that they are in contact in the vertical and horizontal direction
μ	dynamic fluid viscosity
ϱ_f	fluid density
$\varphi(\varepsilon)$	ratio h/R at some porosity and some structure of the type A or B

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